

Insert School Logo

**Semester One
Examination 2020
Question/Answer booklet**

**MATHEMATICS
METHODS UNIT 1**

**Section One:
Calculator-free**

Student Name: _____

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction tape/fluid, erasers, ruler, highlighters

Special Items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

	Number of questions available	Number of questions to be attempted	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	8	8	50	52	35
Section Two Calculator—assumed	17	17	100	98	65
				150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.
2. Answer the questions according to the following instructions.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Section One: Calculator–free**35% (52 marks)**

This section has **eight (8)** questions. Attempt **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 50 minutes

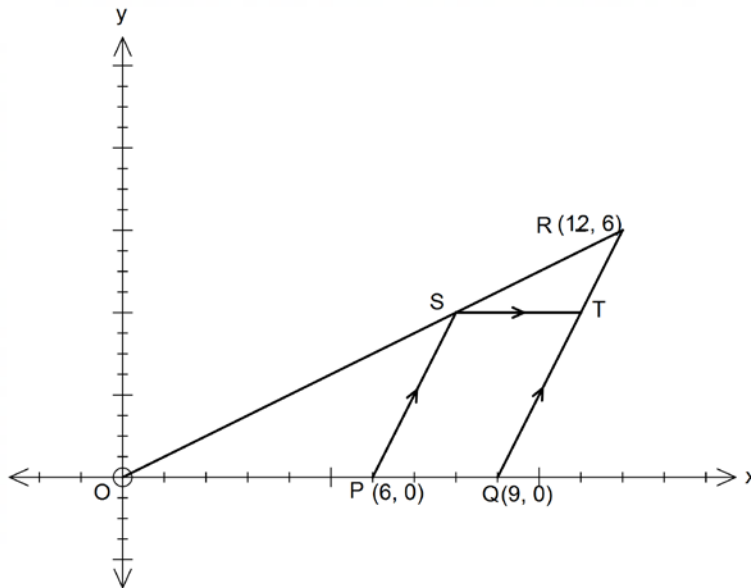
Question 1 (4 marks)

(a) If $\sin 20^\circ = p$, determine $\sin 340^\circ$ in terms of p . (1 mark)

(b) Given that $\angle A + \angle B = 90^\circ$ and $\sin A = \frac{5}{13}$, evaluate $\cos (A - B)$. (3 marks)

Question 2 (11 marks)

In the diagram P, Q and R are the points (6, 0), (9, 0) and (12, 6) respectively. Line PS is parallel to QR. Line ST is parallel to the x-axis. The equation of line OR is given by $x - 2y = 0$.



(a) Show that the equation of PS is $y = 2x - 12$.

(3 marks)

$$m_{RQ} = \frac{6-0}{12-9} = \frac{6}{3} = 2$$

$$m_{RQ} \parallel m_{SP}$$

$$m = 2, (6, 0)$$

$$y = mx + c$$

$$0 = 2(6) + c$$

$$c = -12$$

$$\therefore y = 2x - 12$$

(b) Find the coordinates of point S.

(2 marks)

$$\begin{array}{rcl} x - 2y = 0 & \text{---} & \textcircled{1} \\ 2x - 12 = y & \text{---} & \textcircled{2} \\ x = 2y & \text{---} & \textcircled{1} \end{array}$$

Sub ① into ②

$$\begin{array}{rcl} 2(2y) - 12 = y & & \\ 4y - 12 = y & & \end{array}$$

$$\begin{array}{rcl} 3y = 12 & & \\ y = 4 & & \\ x = 8 & & \end{array}$$

$$S(8, 4)$$

(Question 2 continued)

(c) Find the coordinates of point T.

(3 marks)

$$R(12,6) \quad Q(9,0)$$

$$m_{RQ} = \frac{6-0}{12-9}$$

$$= \frac{6}{3}$$

$$= 2$$

$$(12,6) \quad m=2$$

$$y = mx + c$$

$$6 = 2(12) + c$$

$$c = -18$$

$$\therefore y = 2x - 18$$

$$\text{Alt } y = 4$$

$$4 = 2x - 18$$

$$22 = 2x$$

$$x = 11$$

$$T(11,4)$$

(d) State the simplified ratio of PS : TR

(3 marks)

$$\text{Distance PS } (6,0) \quad (8,4)$$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(6-8)^2 + (0-4)^2}$$

$$= \sqrt{4+16}$$

$$= \sqrt{20}$$

$$\sqrt{20} : \sqrt{5}$$

$$2\sqrt{5} : \sqrt{5}$$

$$2 : 1$$

$$\text{Distance TR } (11,4) \quad (12,6)$$

$$d = \sqrt{(11-12)^2 + (4-6)^2}$$

$$= \sqrt{1+4}$$

$$= \sqrt{5}$$

Question 3 (4 marks)

Daniel decides to plant trees as part of his #climateaction project. He has three jarrah trees and two paperbark trees. On each of the next three days, Monday, Tuesday and Wednesday, Daniel selects one tree at random to plant.

Find the probability that Daniel

- (a) selects a jarrah tree to plant on Monday. (1 mark)

$$\frac{3}{5} \checkmark$$

- (b) plants the same kind of tree on all three days. (1 mark)

$$\begin{aligned} & P(JJJ) + P(PPP) \\ & \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{2}{5} \times \frac{1}{5} \times \frac{0}{5} \\ & \frac{6}{60} + 0 = \frac{6}{60} \checkmark = \frac{1}{10} \end{aligned}$$

- (c) does not plant the same kind of tree on consecutive days. (1 mark)

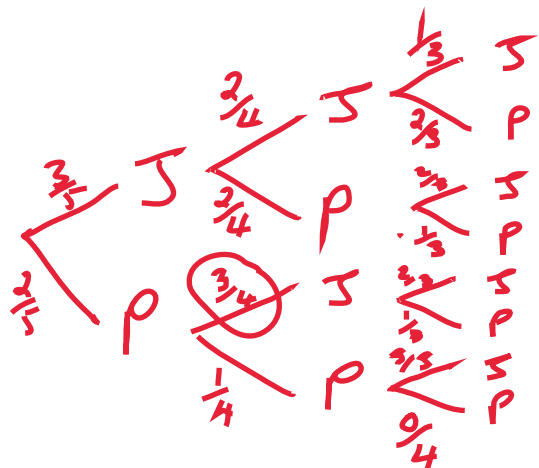
$$\begin{aligned} & P(JPJ) \text{ or } P(PJP) \\ & = \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} \\ & = \frac{12}{60} + \frac{6}{60} = \frac{18}{60} \checkmark = \frac{3}{10} \end{aligned}$$

- (d) plants a Jarrah tree on Tuesday, given that he planted a Paperbark on Monday. (1 mark)

$$P(J \text{ Tues} \mid P \text{ Mon})$$

$$= \frac{P(J \text{ Tues and } P \text{ Mon})}{P(P \text{ Mon})}$$

$$= \frac{\frac{2}{5} \times \frac{3}{4}}{\frac{2}{5}} = \frac{3}{4}$$



Question 4 (4 marks)

Consider the graph with the equation $g(x) = -x^3 - 2x^2 + 7$.

- (a) State the degree of the polynomial $-x^3 - 2x^2 + 7$. (1 mark)

3 ✓

- (b) Describe the behaviour of $g(x)$ when $x \rightarrow \infty$. (1 mark)

$g(x) \rightarrow -\infty$ ✓

- (c) The graph of $g(x)$ is translated 3 units to the right and then dilated vertically by factor 2. What is the y -intercept of the resulting graph? (2 marks)

if quadratic

$y = 2(x-3)^2$

$g(x) = 2(- (x-3)^3 - 2(x-3)^2 + 7)$

At $x=0$ $g(x) = 2(+27 - 18 + 7) = 2(16) = 32 \therefore (0, 32)$

$\frac{2 \times 34 - 18}{16}$

Question 5 (3 marks)

Given that $x = 1$ is a solution to the equation $x^3 - 2x^2 - 5x + 6 = 0$, find the remaining solutions. (3 marks)

$x^3 - 2x^2 - 5x + 6 = (x-1)(ax^2 + bx + c) + r$

$a = 1$

$-2x^2 = bx - ax^2$

$-2 = b - 1$

$b = -1$

$-5x = cx - bx$

$-5 = c + 1$

$c = -6$

$6 = -c + r$

$6 = -(-6) + r$

$r = 0$

OR $x^2 - x - 6$

$x-1 \overline{) x^3 - 2x^2 - 5x + 6}$

$-(x^3 - x^2)$

$-x^2 - 5x$

$-(-x^2 + x)$

$-6x + 6$

$-(-6x + 6)$

0

$(x-1)(x^2 - x - 6)$

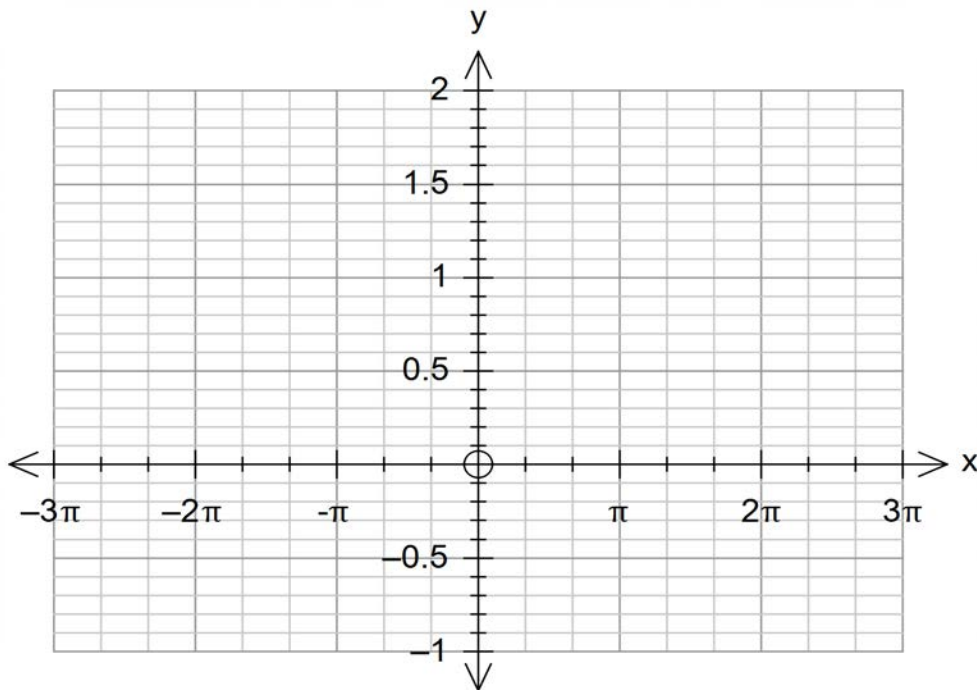
$(x-1)(x-3)(x+2) \therefore$ Solns $x = 1, 3, -2$

See next page

Question 6 (14 marks)

(a) Determine the amplitude and period of the function given by $y = 1 - \cos\left(\frac{x}{2}\right)$. (2 marks)

(b) Sketch the graph of $y = 1 - \cos\left(\frac{x}{2}\right)$ on the axes below where $x \in [-3\pi, 3\pi]$. (3 marks)



(c) The graph of $y = 1 - \cos\left(\frac{x}{2}\right)$ undergoes the following transformations:

A vertical translation of 1 unit down and then a horizontal translation of $\frac{\pi}{3}$ units to the right.

(i) State the equation of the transformed graph. (2 marks)

(ii) Sketch this graph on the same set of axes above. (3 marks)

(Question 6 continued)

(d) Using the graphs, or otherwise, solve the following equations.

(i) $-\cos\left(\frac{x}{2}\right) = 0$ where $x \in [-3\pi, 3\pi]$. (2 marks)

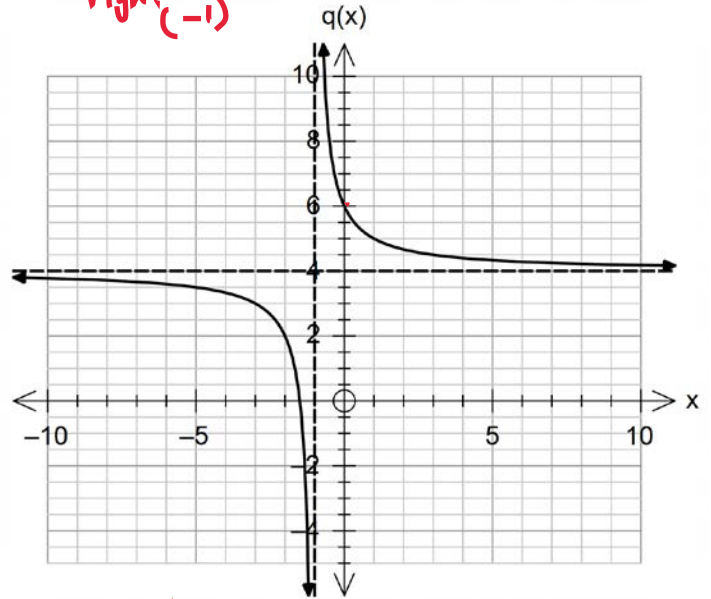
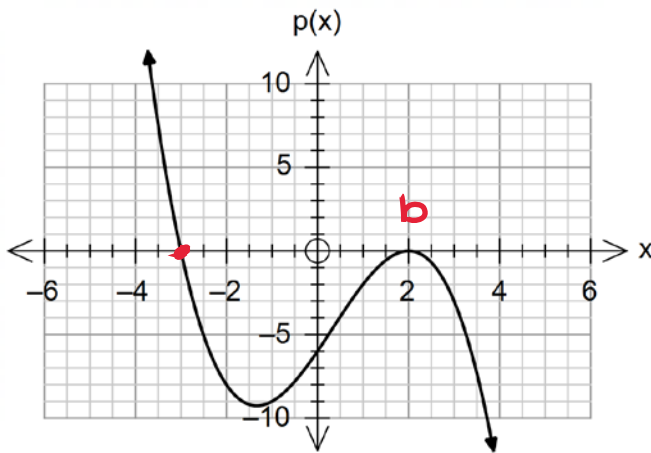
(ii) $3 - 3\cos\left(\frac{x}{2}\right) = 0$ where $x \in [-3\pi, 3\pi]$. (1 mark)

(e) State the function $y = 1 - \cos\left(\frac{x}{2}\right)$ as a sine function in terms of x . (1 mark)

Question 7 (9 marks)

The graphs of $p(x) = a(x - b)^2(x - c)$ and $q(x) = d + \frac{e}{x - f}$ are shown below:

up 4
right (-1)



(a) Find the values of a, b, c, d, e and f .

$d = 4$ ✓
 $f = -1$ ✓ (6 marks)

$b = 2$ ✓
 $c = -3$ ✓

$q(x) = 4 + \frac{e}{x + 1}$

$(0, 5) \quad y = a(x - 2)^2(x + 3)$ ✓
 $-6 = a(4)(3) \quad a = -\frac{6}{12} = -\frac{1}{2}$

$(0, 6) \quad 6 = 4 + \frac{e}{1}$
 $e = 2$ ✓

(b) State the equation of the vertical asymptote of $q(x)$.

(1 mark)

$x = -1$

(c) State the domain of $p(x)$.

(1 mark)

$\{x \mid x \in \mathbb{R}\}$

(d) State the range of $q(x)$.

(1 mark)

$\{y \mid y \in \mathbb{R}, y \neq 4\}$

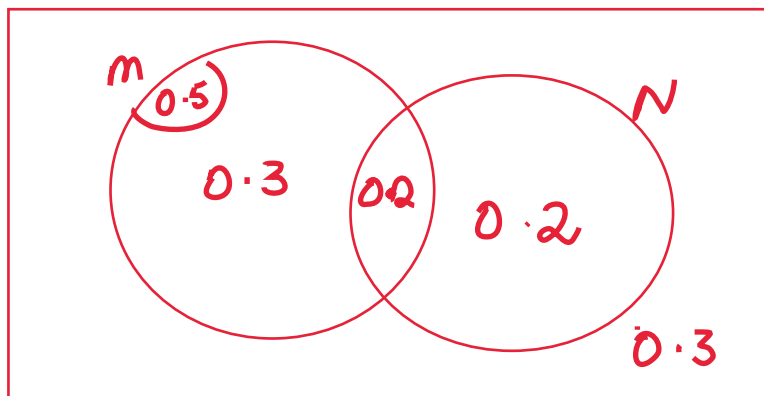
Question 8 (3 marks)

The following information is given:

- $P(M) = 0.5$
- $P(M \text{ and } N) = 0.2$
- $P(\bar{N}) = 0.6$

Draw a Venn diagram to represent this information and determine $P(M \cup N)$. (3 marks)

0.7 ✓



✓✓

End of Questions

Additional working space

Question number(s):